TECHNOLOGY OF SYNERGY REVEALING IN TEACHING OF MATHEMATICS

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ABSTRACT

In present article the possibilities of synergy revealing of mathematical education in secondary school on the basis of modern achievements in science adaptation are investigated. The technology is based on the study of "problem zones" of the development of school mathematics with synergetic effects manifestation on the basis of visual modelling of founding environment using computer and mathematical resources. The technology of modern achievements in science adaptation to school mathematics (chaos "area" of lateral surface of Schwarz cylinder, fuzzy sets and fuzzy logic, fractal geometry, coding theory, etc.) on the basis of phased mathematical modelling and computer-aided design with the manifestation of nonlinear synergetic effects is developed. The founding cluster of generalized construct of modern knowledge, consisting of 4 implementation stages: *initial level* of the essence development of generalized construct for intuitive visual level, *functional stage* of awareness and correction of the features, options, and terms of limit process, *operational stage* of awareness and generality of temporal and functional sequence of learning activity of generalized essence of the construct, *assessment stage* of empirical verification of results, quantitative and qualitative analysis of teaching actions by means of mathematical modelling and computer-aided design, *integrative stage* aimed at the ability to translate the situation of entity's development into the processes of modelling, generalization and transfer. Each stage is integrated with two spirals of founding by means of processes equipment of essence deployment for generalized construct: motivation and applied maintenance of essence development, mathematical and computer modelling of synergetic effects manifestation and attributes.

Keywords: founding, teaching of mathematics, synergy of education

INTRODUCTION

The problem of student's personality development in the process of learning mathematics determines the need to include in a single integrity the processes of self - organization of cognitive activity on the basis of motivational, featured and emotional-volitional, research and meta cognitive, social and personal behaviour strategies. It creates the precedent of person’s expansion and deepening of experience on the basis of his current state, formation and development of intellectual operations and abilities. It will be supported on the basic mechanisms and visual modelling of manifestation and correction opportunities of functional,

operational and instrumental competences in mathematics learning [1]. At the same time, there is the possibility of adaptation of modern achievements in science to school mathematics and computer interactive interaction with mathematics in an open and rich information and educational environment. It will enhance the developmental of effect and educational motivation, reveal connections with real life and practice, create a phenomenon of synergetic effects in the development of complex mathematical knowledge. However, real life puts before higher school graduates the professional problems and preferences based on rapidly breaking into science, economics, communications, and production of innovation systems that require a new quality of ownership of generalized content of school mathematics. Moreover, such innovations as a rule are associated with use of information technologies and require a certain level of intellectual operations development: modelling, associations, analogies, generalization, abstraction, etc. In the economy and the production widely used elements are fuzzy logic, fractal geometry, coding and encryption of information, neural networks, and stochastic processes, nonlinear dynamics, etc. Now a graduate of the Western school has a small opportunity to enroll in prestigious universities and in the best case is forced to educate himself to obtain successful life-career. These trends have also affected the Russian mathematical education to a certain extent: the lower limit of the national exams score dropped to 20 points in 2014, and in the recent years it has been kept at a low level of 27 points. In recent years our always leading teams of students in international mathematical Olympiads do not rise above the 7th place, and in 2017 took the 11th place, giving up places to the teams of China, Singapore, USA, Vietnam, South Korea and other countries. At the same time, young people have more opportunities to identify and realize their abilities, express themselves and self-actualize in educational and professional activities, have become more open to communication and choice of life situations. The younger generation has become more intolerant to dogmatism manifestations, lack of flexibility in training influences, has become pragmatic and consciously assessing personal preferences and possibilities for improvement in the prediction of their future life. These trends show to the teacher increased demands to improve their knowledge of modern content of mathematics and the development of mathematical and computer modelling, used in other sciences. At the same time, the key aspect of the phenomenon of synergetic effects manifestation in learning mathematics on the basis of adaptation of modern achievements in science is the ability to update the stages and characteristics of complex mathematical knowledge essence, phenomena and procedures in the context of the deployment of individual educational routes of students [2]. Thus, the present research is an attempt to develop the technology of adaptation of modern achievements in science to school mathematics with the manifestation of synergetic effects during the deployment of individual educational routes in learning mathematics in resource classes by means of research of multi-stage mathematical and information tasks [3].

MATERIALS AND METHODS

The founding of personal experience becomes especially actual in the modern period when the tendencies to motivational sphere development, meta cognitive experience, processes of self-actualization and self-realization of the person are growing. It is realized in context of the deployment of adequate pedagogical conditions, subject contents, means, forms and technologies of training to subjects of natural science and humanitarian cycles increase. Research and adaptation to school or university mathematics of modern achievements in science are vividly and significantly presented in applications to real life, the development of other sciences. High technology and manufacturing can be an effective tool for the development of complex knowledge based on the founding of personal experience. *Especially such procedures are shown at research and adaptation to school mathematics of difficult mathematical knowledge by step-by-step and multifunctional manifestation of its generalized essence and its integration with school educational elements – these in our work are modern achievements in science*. Since the essence reveals of its reality in the totality of external characteristics of the object, revealing the essence through the philosophical categories of the internal, general, content, cause, necessity and law, it becomes possible to determine *the component composition of the content and procedural characteristics of the manifestation of the essence* [5]. It reveals the content modus: sign symbolic, verbal, figurative-geometric and tactile-kinesthetic manifestations; procedural modus: historical-genetic, specific - activity, experimental and applied manifestations. This variability and mobility of the subject matter requires updating of step-by-step progress to its cognition and defines the third dimension of the essence-personality-adaptive in its characteristics. It defines the three-component integrity of the subject matter as an object of cognition in the course of cognitive activity.

Technology of synergy revealing in mathematics education

Adaptation processes are considered by psychologists and teachers as a dynamic complex of integral interaction of internal results (system of knowledge, skills, attitudes, values) and adequate mechanisms of adaptation of the personality to changes in the environment and the results of activities with developmental effect. Initially the phenomenon of adaptation of modern achievements in science (as manifestations of the environment) to school mathematics in the context of updating the mechanisms of adaptation and teaching of the personality acts as a process and the cognitive result of the unclear, uncertain state of generalized construct of the essence and its individual qualitative manifestations. The following figure 1 presents a graph of stages coordination of the essence manifestation of modern scientific knowledge in the mathematics development and the stages of synergy manifestation of mathematics education [6].

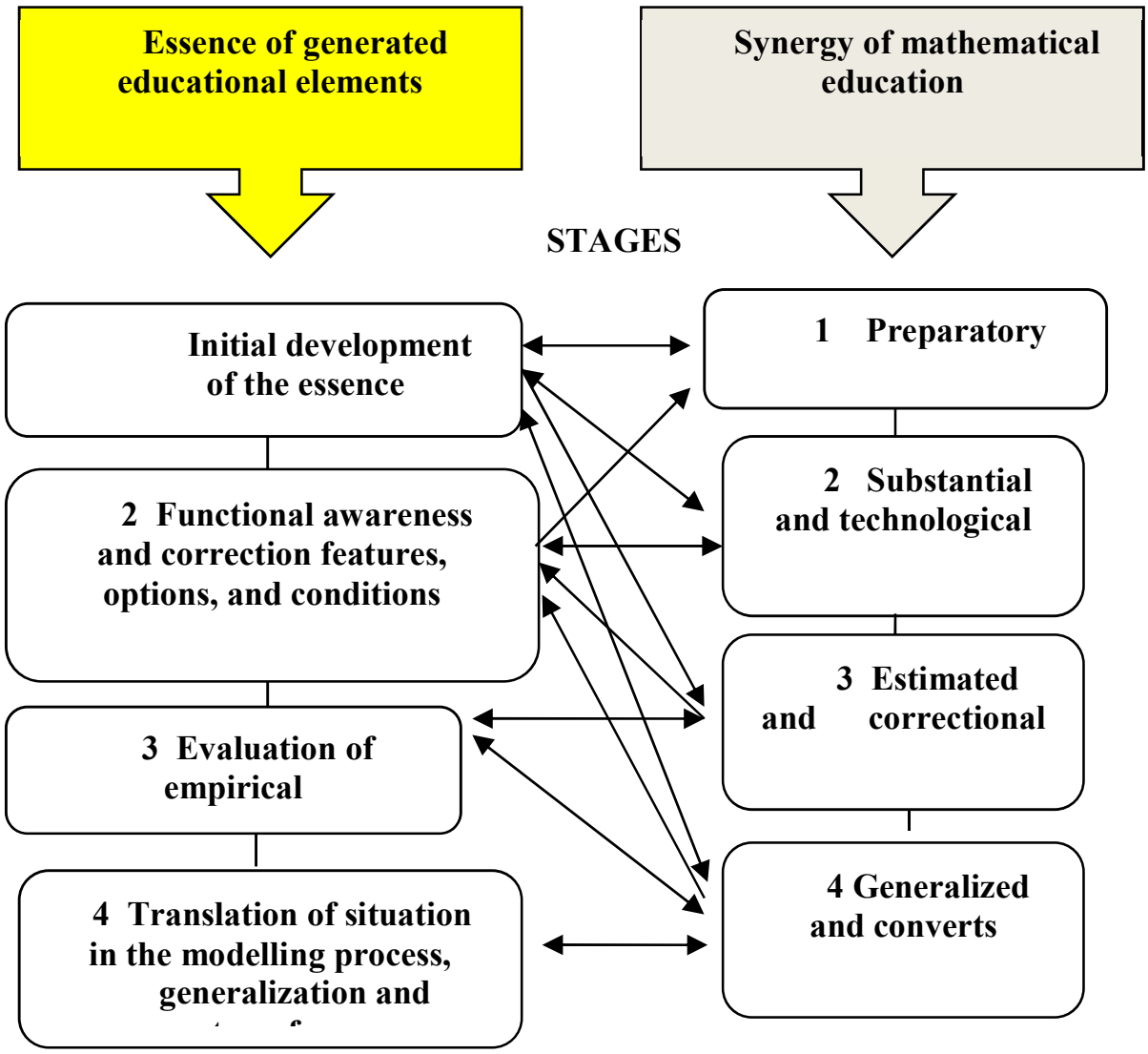

Fig.1. Stages coordination of the essence manifestation of modern scientific knowledge and synergy manifestation of mathematics education

Components of generalized construct adaptation of “problem zones” to the contents of school mathematics:

- ***The creation of a motivational field***: visual modelling *(lessons-lectures, videos, project activity, presentations, business games)* of motivational - applied situations of "problem zone" development in mathematical education; standards and samples of methods and means using which adequate to a problem with detailing, analysis and features; presentation of research stages, methods and procedures, historical and genetic and problem justification of emergence and applications of generalized construct of modern knowledge in the context of "problem zone" development; increased attention to development and manifestations of thinking criticality trained in processes of self-analysis and reflection of pedagogical processes; formation of stable motives of search and development of new in mathematical and information activities; expansion and development of database of scientific data and a set of scientific research methods on the basis of school subject; multiple experience of micro

problems solving in the mode of “warming up” and the development of up situational activity (emotional experience, reflection, visual modelling, insight, verification of solutions, transfer); willingness to debate and multiplicity of problem solutions; identification and promotion of creative behaviour samples and its results). This phase corresponds to phase 1-2 and is adequately implemented in 10-12 activities of classroom or extracurricular activities.

- ***Setting of multiple, multilevel and polyvalent tasks in the field of "problem zone"*** to update the learning of qualitative and quantitative characteristics and parameters of "problem zone" (a variety of approaches and methods, variation of parameters and content structures, the singularity of the results forecast and the integrity of tools used), as well as the deployment of individual educational trajectories for small groups of schoolchildren (determination of the composition and direction of small groups, distribution of roles), selection and actualization of practice-oriented research activities on the stages of underlying procedures development for the essence identifying of generalized structure of modern knowledge and their adaptation to school mathematics: to investigate real functionality by means of computer and mathematical modelling, operability and applied context of founding processes of modern knowledge development in the dialogue context of mathematical, information, natural cultures. *This phase corresponds to phase 1-4 and is adequately implemented in 10-12 activities of classroom or extracurricular activities.*
- ***Multiple goal-setting of research processes of generalized construct of " zone of modern achievements in science”:*** creation of the plan of problem solving in conceptual, subject, information and mathematical models; possibilities of ICT-support tools analysis; identification of stages content of the essence founding of generalized construct; formalization, genesis of history, presence of the essence manifestation samples of reference and situational levels; intuition and prediction of results, search and algorithm solutions; insight, fixation and verification of procedures and algorithms; presentation of results; creating situations of intellectual effort and self-organization of learners, updating of uncertainty and bifurcation points of mathematical procedures; ability to adapt and develop in social communication on the basis of cultures dialogue; the variation of conditions and the data of problem; taking into account the probable and improbable circumstances, evaluation of their effectiveness. *This phase corresponds to the steps 1-4 and adequately implemented in classroom or extracurricular activities.*
- ***Founding cluster of modern knowledge generalized construct*** is a didactic model of the essence founding of generalized construct, consisting of 4 stages implementation: *initial level* of the essence development of generalized construct on an intuitive level, *the functional stage* of awareness and correction of functions, parameters and conditions of generalized construct being, *the operational stage* of awareness and generalization of time and functional sequence of actions to develop of generalized construct essence, *the evaluation stage* of empirical verification, quantitative and qualitative analysis of actions by means of mathematical modelling and computer-aided design, *an integrative*

stage aimed at the ability to transfer the situation of the entity's development into the processes of modelling, generalization and transfer. Each stage is integrated with two spirals of founding of equipment means of deployment processes of generalized construct essence: motivational and applied maintenance of development essence processes and mathematical and computer modelling of synergetic effects and attributes manifestation.

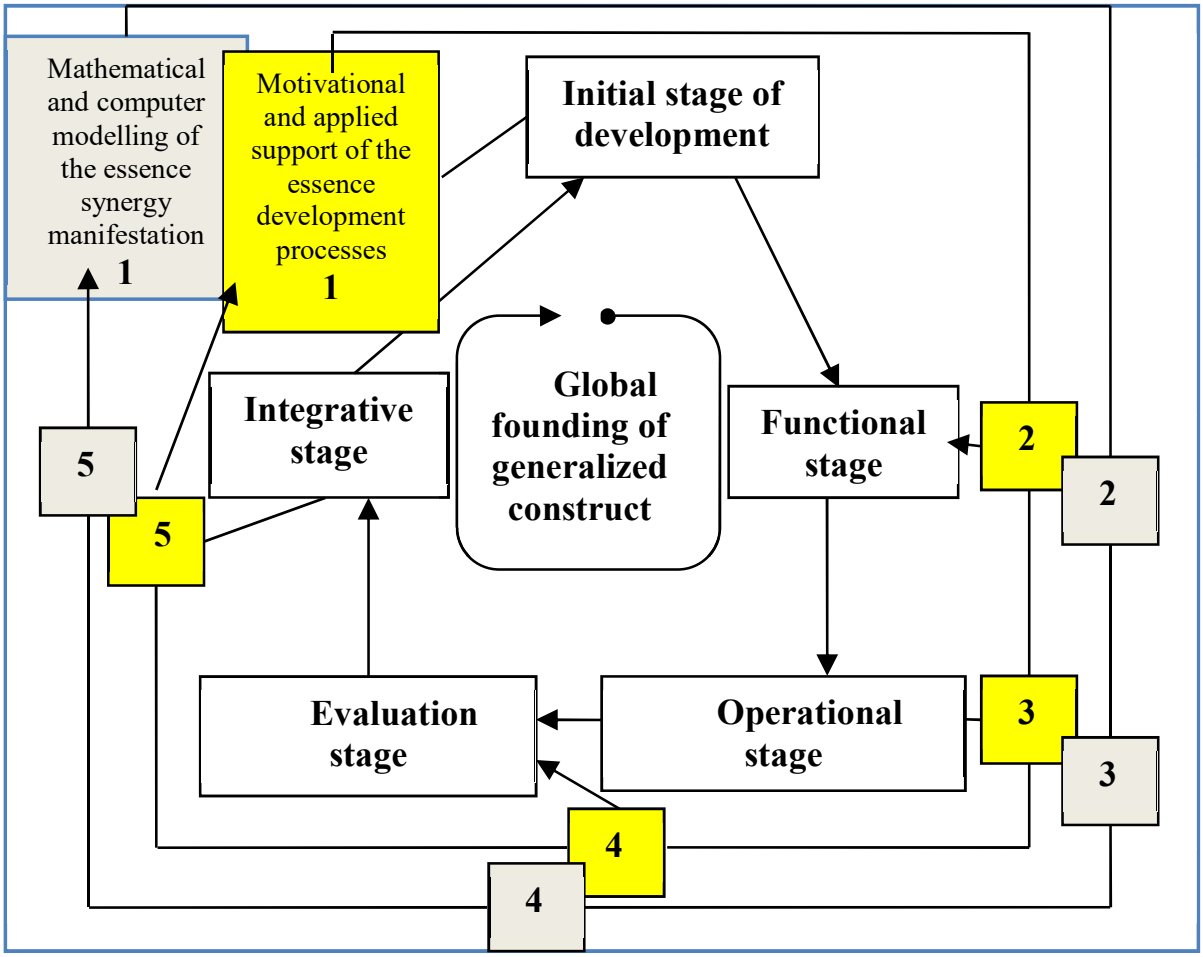

Fig.2. Cluster of the essence founding of modern knowledge generalized construct in teaching mathematics

Decoding of function blocks contents of figure 2 (on the example of generalized concept – function limit [6]):

1 – the area of polyhedral complexes of lateral surface triangulations of regular (layers of same height) cylinder or Schwartz's "boot" [7]; Koch's snowflake, Sierpinski napkin (perimeter and area as the limiting constructs) [8]; attractors and basins of attraction of piecewise-linear maps; multiple homothetic of the plane and space (fixed point, polar, basins of attraction) [9].

Example 1. "Area" pathological properties of a lateral surface of Schwartz's cylinder are well studied in a so-called "regular" case (see for example [10]). It occurs when its height of H breaks to m equal parts (respectively – cylinder layers) and the circles lying in the basis are divided

to n of equals parts with the subsequent shift on φ each layer on π/n. At such triangulation of lateral surface of the cylinder the formula for calculation of its "area" by means of the turned-out polyhedrons at m, n →∞ has an appearance:

$$S_q = 2\pi R\sqrt{R^2\frac{\pi^4}{4}q^2 + H^2}, \quad (1)$$

where $q = \lim_{m,n\to\infty}\frac{m}{n^2}$ and S_q is a lateral surface of the cylinder for such triangulation.

Thus "area" of a lateral surface of Sq of the regular Schwartz's cylinder of height of H and radius of R (if this limit exists – final or infinite value) completely is defined by a limit q. It is clear that true value of the area of a lateral surface (q = 0) can be received by consideration of the tangent planes in points of a triangulation and the subsequent transition to a limit of the areas of external polyhedrons at unlimited of crushing splitting. In article of E.I. Smirnov and A.D. Uvarov [5] the behaviour of function (1) and a corner α between triangles with the general basis is investigated if $m = f^n(a_0)\cdot n^2$ and m, n →∞ , where

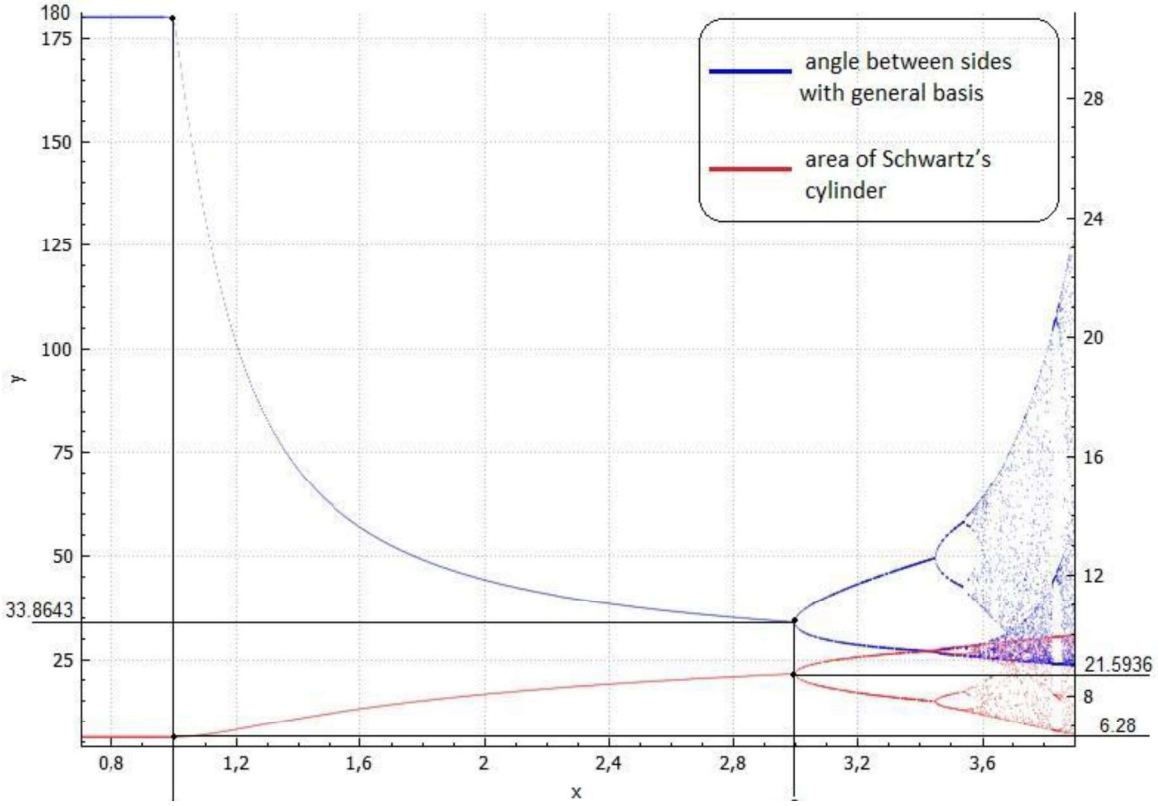

Fig.3. The bifurcation diagram of "area" and angles of Schwartz's cylinder

$f(a_0) = xa_0(1-a_0)$ – the logistic mapping adequate to P. Verhulst's scenario [8]. Authors received the following bifurcation diagram (Fig. 3) with use of information technologies (Qt Creator environment).

1 – T. Malthus's logistic equation, P. Verhulst's script; fractal geometry, Julia sets and Mandelbrot sets (history, mathematical and computer modelling, applications) [11].

2 - partial limits, covering theorem, upper and lower function limits; area of polyhedral complexes of lateral surface triangulations of an irregular (layers of different heights) cylinder or Schwartz's "boot"; multiple homothetic of the plane and space in dynamic chaos (Serpinsky triangle, Cantor set, Menger's "sponge").

Example 2. Let us consider the coordinate of point O(1,1) and coordinates of vertices of regular triangle ABC: A(0,0), B(1,0), C($\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$). Homothetic ⟨, ®, © — with the coefficient k = 0,5 and centers A, B, C respectively and iteration process of infinite sequence of points construction (orbits) are considered. Homothetic f $M = \{⟨, ®, ©\}$ of point x_0 on n-step are selected with probability p = 1/3 and constructed the image $x_{n+1} = f(x_n)$ of point x_n (Fig. 4).

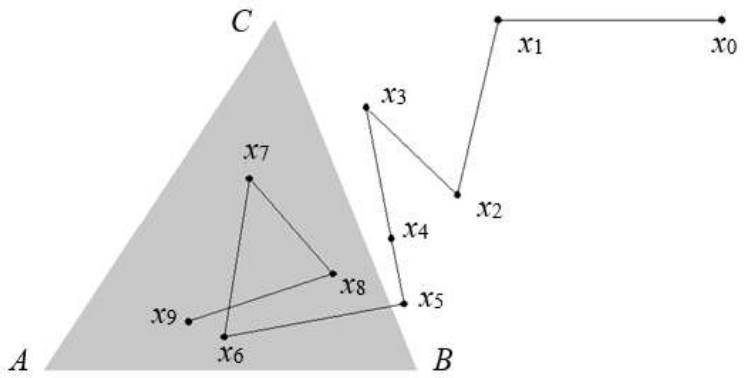

Figure 4. Iteration process of attractor construction

The numerical experiment shows that the orbit of an arbitrary point tends to the Serpinsky's triangle (Fig. 4). Since the transformation f is random in each iteration, so any two orbits with the same starting point x_0 do not coincide, any orbit is random, its behaviour is unpredictable (even in the first iteration). This property is a necessary sign of the chaotic dynamic system. Note that fractal dimension $\dim_M F$ of Serpinsky's triangle is a fractional number $\log_2 3$ [11].

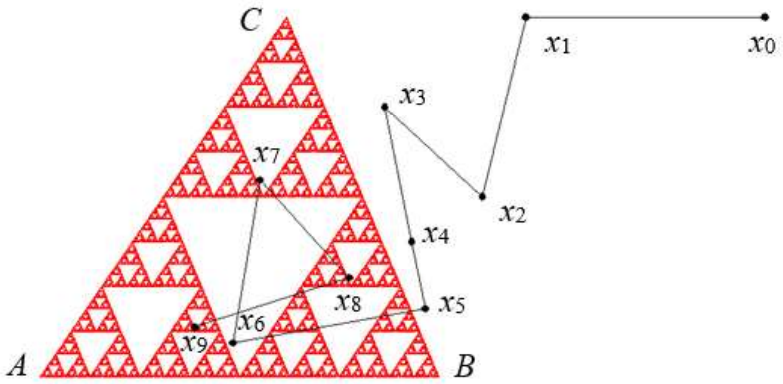

Figure 5. Serpinsi's triangle as an attractor of iteration

2 - tree and Feigenbaum's constant and the transition from order to chaos; fractal structure of Van der Varden's function (computer and mathematical modelling, curve approximations, continuity and no differentiability of the curve) [12].

3 – computer simulation of ε-δ-Cauchy language; business game "Finding of min N(ε) for rational sequences"; variation of parameters and computational design of spatial limit of a sequence [13]; computer design and fractal variations of Julia sets, sets and Mandelbrot's sets (iteration, fixed point, variation of the polynomial n-th degree, basins of attraction); study of attractors of nonlinear mappings (Bernoulli, Henon, display "Baker", Arnold's "cat", tent-like function) [14].

3 – Lorenz's and Henon's strange attractors; affine transformations and Barnsley's maple leaf; Sierpinsky's dust and art fractals.

4 – computer design and mathematical modelling of point's neighborhood on a plane and in space for various metrics, universality of point convergence and Euclidean metric; numerical methods for area finding of a curvilinear trapezoid (rectangles, trapezoids, Simpson methods).

4 – computer and mathematical modelling: Hutchinson's transformation, ISF method (Iterated Function Systems), multifractals, limit in Hausdorff's metric.

5 – computer and mathematical modelling of generalized solution of wave equation; computer design of strange cross-attractors of affine plane transformation.

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– generalized curves and Dirac's δ-function (instant impact and impulse), generalized functions and limits, summation of divergent series); Lebesgue integral (history, advantages, applications); non-standard analysis by A. Robinson (history, axioms, theorems).

• ***Updating the attributes of synergy (bifurcation, attractors, fluctuations, basins of attraction) in the research process of generalized construct of modern knowledge*** - Forms: distance learning of project teams, laboratory and design classes, multistage mathematical and information tasks, conference, workshops, networking and discussion forums; Tools: mathematical and computer modelling, QT Creator-free cross-platform IDE for development in C++, pedagogical software products, small means of information ClassPad400, WebQuest - as a means of integrating Web-technologies with educational subjects, Wiki-sites, Messenger, Skype, Webinar, TeamSpeak, Discord; Technologies: compliance graphs of mathematical knowledge and procedures, work in small groups, WebQuest – as a technology of self –organization in collective creativity , project method, Wiki-technology, visual modelling, founding of experience. This phase corresponds to the steps 1-4 and adequately implemented in the activities of classroom or extracurricular activities.

• ***Effective dialogue of mathematical, information, natural-science and humanitarian cultures*****:** process of synergy manifestation of knowledge and procedures is implemented in stages according to selected levels of cultures dialogue actualization in the direction of basing didactic procedures deployment. Equipment and development of generalized construct essence of «modern achievements in science zone" and obtaining probabilistically guaranteed results are presented:

- structural and logical level of knowledge and procedures integration of various disciplines in the context of dialogue and unity of cultures multiculturalism in students development;

- level of actualization of the unity and characteristics of cultures dialogue in the diversity of intercultural communication in productive development of deployment stages of generalized construct essence;

- level of self-organization and self-development of intercultural interactions in the context of generalized essence updating.

This phase corresponds to the steps 1-4 and adequately implemented in the activities of classroom or extracurricular activities.

• ***Forecast and "extra - products" of research (video clips, design methods, computer-aided design and intelligent systems, web quests, artistic and graphic creativity, presentations):***

- history, constructing using intelligent environments and cultures dialogue, computer design, mathematics of "extra-product" learning of

generalized construct (Menger's "sponge", smooth Julia sets, electronic signature, non-standard analysis by A. Robinson (history, axioms, theorems), etc.); building of first iterations of Menger's "sponge" origami and multifractal composition, etc. ; finding of topological and fractal dimension and properties of Menger's "sponge, presentation of natural and industrial effects, which implement the essence of generalized construct: a computer simulation of the transition layer of solid solutions series, fractal sculptures and architectural masterpieces; dynamic cross-section of fractal objects (video clips);

- Minkovsky's curve and Harter's "dragon" - history, construction, computer graphics, topological and fractal dimensions, natural analogues and computer modelling, generators of "dragon" contour with a variable angle;

- stochastic fractals and modelling of natural phenomena and processes: image of planets, satellites, clouds and mountain ranges; method of random movement of midpoint; modification of fractals with different generators.

This phase corresponds to all stages 3-4 and is adequately implemented in the activities of classroom or extracurricular activities.

CONCLUSION

Identification and investigation of "zones of modern achievements in science" in teaching mathematics by means of computer and mathematical modelling allows mastering generalized constructs of basic educational elements in the context of synergetic effects, cultures dialogue and knowledge integration from different fields of science. At the same time, the openness of educational environment, complexity of mathematical structures, plurality of goal-setting and possibility of "extra-product" obtaining create the basis for the effective development of intellectual thinking operations, increasing of educational and professional motivation, creativity and self-organization of students in the context of intercultural communications. In accordance with identified attributes of modern achievements in science adaptation to school mathematics can be investigated such "problem zones": elements of fractal geometry in the context of self-organization and self-similarity processes of geometric objects and functional dependencies, Schwartz's cylinder in context of the essence of surface area identifying, cellular automaton, coding and encryption of information, chaos and catastrophe theory, fuzzy sets and fuzzy logic, etc.

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